

November 13

Completing the Square with the Equation of a Circle

Determine the center and radius of:

EX1) $x^2 + y^2 + 14x - 12y + 60 = 0$

Step 1: Get variables on one side of equal sign and plain number on the other.

$$x^2 + y^2 + 14x - 12y + \begin{matrix} 60 \\ -60 \end{matrix} = 0$$

$$x^2 + y^2 + 14x - 12y = -60$$

Step 2: Rearrange to group x's and y's together.

$$(x^2 + 14x) + (y^2 - 12y) = -60$$

Step 3: Make empty shapes to complete the square in.

$$(x^2 + 14x + \boxed{49}) + (y^2 - 12y + \boxed{36}) = -60 + \boxed{49} + \boxed{36}$$

Step 4: Fill in shapes. Take number in front of x and y. Divide it by 2. Then square it.

$$\frac{14}{2} = (7)^2 = 49 \quad \frac{-12}{2} = (-6)^2 = 36$$

Step 5: Build parentheses. Combine plain numbers.

$$(x + 7)^2 + (y - 6)^2 = 25$$

$$\text{center: } (-7, 6)$$

$$\sqrt{r^2} = \sqrt{25}$$
$$\boxed{r = 5}$$

$$\text{Ex 2 } x^2 + y^2 - 4x - 8y - 55 = 0$$

$$x^2 + y^2 - 4x - 8y = 55$$

$$(x^2 - 4x) + (y^2 - 8y) = 55$$

$$(x^2 - 4x + 4) + (y^2 - 8y + 16) = 55 + 4 + 16$$

$$\frac{-4}{2} = (-2)^2 = 4$$

$$\frac{-8}{2} = (-4)^2 = 16$$

$$(x - 2)^2 + (y - 4)^2 = 75$$

$$\text{center: } (2, 4)$$

$$\sqrt{r^2} = \sqrt{75}$$

$$r = 5\sqrt{3}$$

$$\begin{array}{l} \sqrt{75} \\ \swarrow \searrow \\ 5 \quad 15 \\ \swarrow \searrow \\ 5 \quad 3 \\ \sqrt{3 \cdot 5 \cdot 5} \\ 5\sqrt{3} \end{array}$$

$$\text{Ex 3 } x^2 + y^2 + 6x - 9 = 0$$

$$x^2 + y^2 + 6x = 9$$

$$(x^2 + 6x) + y^2 = 9$$

$$(x^2 + 6x + 9) + y^2 = 9 + 9$$

$$\frac{6}{2} = (3)^2 = 9$$

$$(x + 3)^2 + y^2 = 18$$

$$\text{center: } (-3, 0)$$

$$\sqrt{r^2} = \sqrt{18}$$

$$r = 3\sqrt{2}$$