October 30

Vertical Asymptotes and Holes of Rational Functions

vertical asymptotes – imaginary "up and down" line that the graph can't cross

hole – a gap in the graph (not seen on calculator screen)

Find VA, hole, and domain of:

Ex1 $f(x) = \frac{x^2-16}{2x^2-7x-4} = \frac{(x+4)(x-4)}{(x-4)(2x+1)}$

Step 1: FACTOR!!

x^2-16

$\sqrt{x^2}=x \quad \sqrt{16}=4$

$(x+4)(x-4)$

$2x^2-7x-4$

$2x^2 \cdot -4 = -8x^2$

$-8x + 1x = -7x$

$\frac{2x^2}{2x} - \frac{8x}{2x} \mid + \frac{1x}{1} - \frac{4}{1}$

$2x(x-4) \mid 1(x-4)$

$(x-4)(2x+1)$

Step 2: Cancel.

Step 3: Look at denominator ONLY!

Factors that cancel make holes.

$x-4=0$

$+4 \quad +4$

hole : $x=4$

Factors that stayed make vertical asymptotes.

$2x+1=0$

$-1 \quad -1$

$\frac{2x}{2} = \frac{-1}{2}$

VA : $x=-\frac{1}{2}$

Holes and vertical asymptotes are not part of the domain.

domain: $x \neq 4, -\frac{1}{2}$

Ex2 $f(x) = \dfrac{(x-1)}{(x-3)(x+1)}$ ← ∞ ∞

holes: none

$x-3=0$ $+3\ +3$

VA: $x=3$

$x+1=0$ $-1\ -1$

VA: $x=-1$

domain: $x \neq 3, -1$

Ex3 $f(x) = \dfrac{x^2-4x}{x-4} = \dfrac{x(x-4)}{(x-4)}$ ← ∞ ∞

$\dfrac{x^2-4x}{x}$ GCF: x

$x(x-4)$

$x-4$ does not factor

$x-4=0$ $+4\ +4$

hole: $x=4$

VA: none

domain: $x \neq 4$