February 11

Solve Polynomial Equations

Fundamental Theorem of Algebra

The degree of the polynomial tells you the number of zeroes (solutions) that it has.

Ex 1 Solve: $x^3 + 3x^2 - x + 12 = 0$ (degree)

step 1: Graph to find real zeroes.

$x = -4$ m: 1

step 2: Use synthetic division to divide out real zeroes.

$$\begin{array}{r|cccc} -4 & 1 & 3 & -1 & 12 \\ & 0 & -4 & 4 & -12 \\ \hline & 1 & -1 & 3 & \boxed{0} \end{array}$$ ← Remainder

$x^2 - x + 3 = 0$

step 3: Solve what is left to find remaining zeroes.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a: 1 b: −1 c: 3

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{-11}}{2}$$

$\sqrt{-11}$ → $\sqrt{\ominus 11}$

$i\sqrt{11}$ (prime)

$$x = \frac{1 \pm i\sqrt{11}}{2}$$

$$\boxed{x = -4, \ \frac{1 \pm i\sqrt{11}}{2}}$$

Ex 2 Solve: $x^4 + 4x^3 - 4x^2 - 36x - 45 = 0$ ← degree

$x = -3$ m: 1 $\quad x = 3$ m: 1

-3 | 1 | 4 | −4 | −36 | −45

	0	−3	−3	21	45
	1	1	−7	−15	(0) ← Remainder

$x^3 + x^2 - 7x - 15 = 0$

3 | 1 | 1 | −7 | −15

	0	3	12	15
	1	4	5	(0) ← Remainder

$x^2 + 4x + 5 = 0$

a: 1 $\quad$ b: 4 $\quad$ c: 5

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{-4}}{2} \rightarrow \sqrt{64}$$

$i\sqrt{4}$

(2)(2)

$i\sqrt{2 \cdot 2}$

$2 \cdot i\sqrt{\ }$

$2i$

$$x = \frac{-4 \pm 2i}{2} \quad (\div 2 \text{ on } -4,\ 2i,\ 2)$$

$$x = \frac{-2 \pm 1i}{1}$$

$x = -2 \pm i$

$$\boxed{x = -3, 3, -2 \pm i}$$

(Ex3) Solve: $x^6 - 8x^5 - 6x^4 + 200x^3 - 499x^2 + 24x + 720 = 0$ ← degree

$x = -5$ m:1, $x = -1$ m:1, $x = 3$ m:2, $x = 4$ m:2

$\boxed{x = -5, -1, 3, 3, 4, 4}$