January 29

Inverses

An inverse is a reflection over the line $y=x$. The x-values and y-values trade places.

Ex1 Find the inverse of $f(x)=9x+4$

Step 1: If it starts with $f(x)$, change that to a y.

$$y=9x+4$$

Step 2: Make x and y trade places.

$$x=9y+4$$

Step 3: Solve for y.

$$\begin{array}{rcl} x & = & 9y+4 \\ -4 & & \quad -4 \\ \hline \frac{x-4}{9} & = & \frac{\cancel{9}y}{\cancel{9}} \end{array}$$

$$\boxed{\frac{x-4}{9}=y}$$

Ex2 Find the inverse of $f(x)=\frac{6x-3}{7}$

$$y=\frac{6x-3}{7}$$

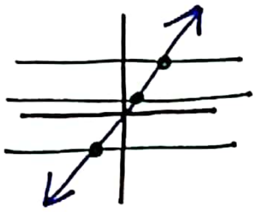
$$7\cdot x=\frac{(6y-3)}{\cancel{7}}\cdot\cancel{7}$$

$$\begin{array}{rcl} 7x & = & 6y-3 \\ +3 & & \quad +3 \\ \hline \frac{7x+3}{6} & = & \frac{\cancel{6}y}{\cancel{6}} \end{array}$$

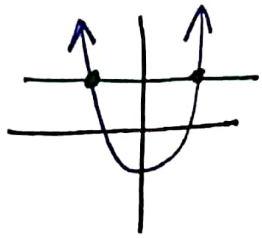
$$\boxed{\frac{7x+3}{6}=y}$$

How do you tell if the inverse is a function?

1. graph the original problem
2. perform Horizontal Line Test (HLT)



passes HLT, inverse is a function



fails HLT, inverse is not a function

Ex3 Find the inverse of $f(x) = 4\sqrt{x}$

passes HLT, inverse is a function

$$y = 4\sqrt{x}$$

$$\frac{x}{4} = \frac{\cancel{4}\sqrt{y}}{\cancel{4}}$$

$$\left(\frac{x}{4}\right)^2 = \left(\sqrt{y}\right)^2$$

$$\boxed{\left(\frac{x}{4}\right)^2 = y}$$

Ex4 Find the inverse of $f(x) = (x+4)^2 - 2$

fails HLT, inverse is not a function

$$y = (x+4)^2 - 2$$

$$x = (y+4)^2 - 2$$
$$+2 \qquad\qquad +2$$

$$\sqrt{x+2} = \sqrt{(y+4)^2}$$

$$\pm\sqrt{x+2} = y + 4$$
$$-4 \qquad -4$$

$$\boxed{\pm\sqrt{x+2} - 4 = y}$$

*when you take a square root to solve, put a $\pm$ in front.

Ex5 Find the inverse of $f = \{(-3, 2), (4, 8)\}$

no repeating y-values means passes HLT, inverse is a function

$$\boxed{\{(2,-3), (8,4)\}}$$

Function Operations

$$(f+g)(x) = f(x) + g(x)$$
$$(f-g)(x) = f(x) - g(x)$$
$$(f \cdot g)(x) = f(x) \cdot g(x)$$

Given $f(x) = 3x+4$ and $g(x) = x-1$, find:

Ex6 $(f+g)(x) = (3x+4) + (x-1)$

$= 3x + 4 + x - 1$

$= \boxed{4x+3}$

Ex7 $(g-f)(x) = (x-1) - 1(3x+4)$

$= x - 1 - 3x - 4$

$= \boxed{-2x-5}$

Ex8 $(f \cdot g)(x) = (3x+4) \cdot (x-1)$

$= 3x^2 - 3x + 4x - 4$

$= \boxed{3x^2 + x - 4}$