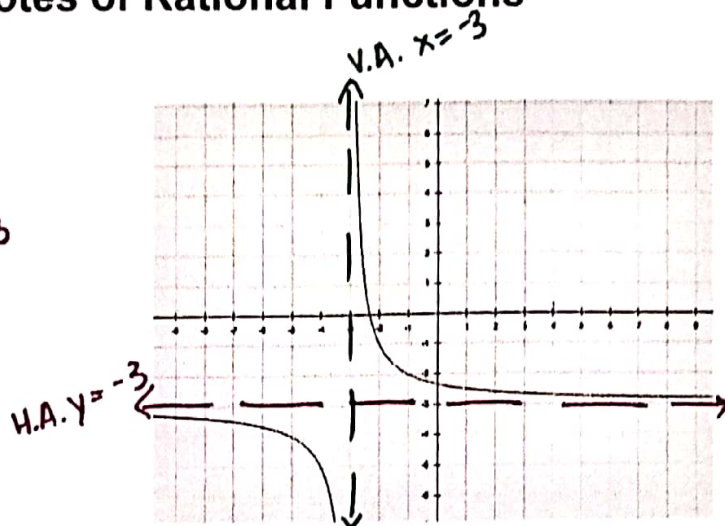
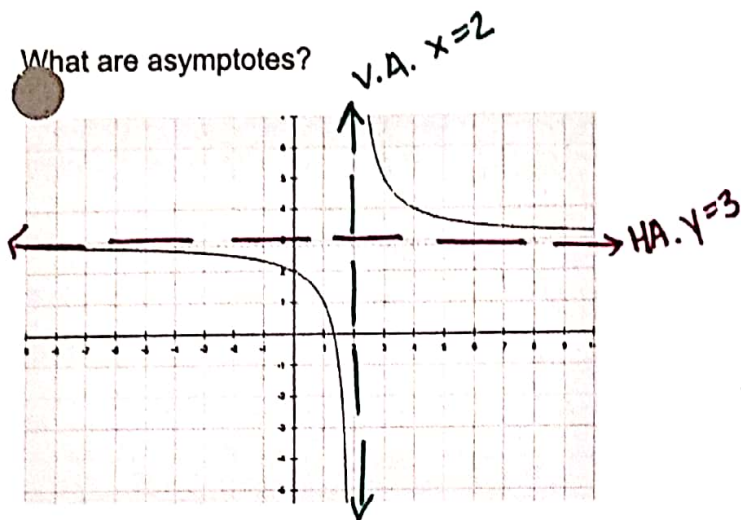


March 11

GUIDED NOTES: Asymptotes of Rational Functions

What are asymptotes?



DOMAIN: The domain of a function is the set of all possible x-values

Vertical asymptotes and holes are values not included in the domain!

Vertical Asymptotes:

To find vertical asymptotes: factor, then cancel. Set factors that stay in denominator equal to 0 and solve.

Q1. $y = \frac{(x-3)}{(x+2)(x-2)}$

$x+2=0 \Rightarrow x=-2$
 $x-2=0 \Rightarrow x=2$

VA: $x=-2$ VA: $x=2$

EX2. $y = \frac{x-1}{x^2+5x+4}$

$y = \frac{(x-1)}{(x+4)(x+1)}$

$x+4=0 \Rightarrow x=-4$
 $x+1=0 \Rightarrow x=-1$

VA: $x=-4$ VA: $x=-1$

$x^2+5x+4 = 4x^2 + 4x + 1x + 4$
 $4x^2 + 4x + 1x + 4 = 4x(x+1) + 1(x+1) = (x+1)(4x+1)$

Holes:

What is the Vertical Asymptote of $y = \frac{x(x-4)}{x-4}$? Graph it, what do you notice?

A HOLE in the graph is when $(x-a)$ is a factor in both the numerator and the denominator (It cancels out!!!)

EX3. $y = \frac{x^2-1}{x^2-2x-3} = \frac{(x+1)(x-1)}{(x-3)(x+1)}$

$x+1=0 \Rightarrow x=-1$ (hole)
 $x-3=0 \Rightarrow x=3$ (VA)

$x^2-1 = (x+1)(x-1)$
 $x^2-2x-3 = -3x^2 + 3x + 1x - 3 = -3x^2 + 4x - 3$
 $-3x^2 + 4x - 3 = -3x^2 + 3x + 1x - 3 = -3x(x-1) + 1(x-1) = (x-1)(-3x+1)$

EX4. $y = \frac{12x+24}{x^2+2x} = \frac{12(x+2)}{x(x+2)}$

$x+2=0 \Rightarrow x=-2$ (hole)
 $x=0$ (VA)

$\frac{12x+24}{x^2+2x} = \frac{12(x+2)}{x(x+2)}$ GCF: 12
 $\frac{12}{x}$

$\frac{x^2+2x}{x(x+2)}$ GCF: x
 $\frac{x}{x+2}$

Horizontal Asymptotes:

To find a horizontal asymptote, we focus on the degree of the numerator and the denominator

step 1: Determine highest exponent in whole fraction.

step 2: Find that term in top and/or bottom.

step 3: Divide those terms.

Examples:

EX5. $y = \frac{6x+5}{3x+9}$

highest: x

$$y = \frac{6x}{3x}$$

HA: $y = 2$

EX6. $y = \frac{4x^3+3x^4-4x^2}{3x}$

highest: x^4

$$y = \frac{4x^4}{0x^4}$$

does not exist

DNE

EX7. $y = \frac{(x-3)(x+4)^2}{(x-5)^3(x+2)^2}$

highest: x^5

$$y = \frac{0x^5}{1x^5}$$

HA: $y = 0$

EVERYTHING ALL TOGETHER:

Find the holes, vertical asymptotes, domain, and horizontal asymptote for each rational function.

EX8. $y = \frac{5x^2+20x}{x^2+11x+28} = \frac{5x(x+4)}{(x+7)(x+4)}$

$\frac{5x^2+20x}{5x} \div \frac{5x}{5x}$ GCF: $5x$
 $5x(x+4)$

$\frac{x^2+11x+28}{x^2-28=28x^2}$
 $7x+4x=11x$
 $\frac{x^2+7x+4x+28}{x(x+7)4(x+7)}$
 $(x+7)(x+4)$

$x+4=0$
 $-4-4$
hole: $x = -4$

$x+7=0$
 $-7-7$
VA: $x = -7$

domain: $x \neq -4, -7$

highest: x^2

$$y = \frac{5x^2}{1x^2}$$

HA: $y = 5$

EX9. $y = \frac{2x^2+10x+12}{x^3+3x^2+2x} = \frac{2(x+3)(x+2)}{x(x+2)(x+1)}$

$\frac{2x^2+10x+12}{2} \div \frac{2}{2}$ GCF: 2
 $2(x^2+5x+6)$
 $x^2-6=6x^2$

$3x+2x=5x$
 $\frac{x^2+3x+2x+6}{x(x+3)2(x+3)}$
 $2(x+3)(x+2)$

$x+2=0$
 $-2-2$
hole: $x = -2$

VA: $x = 0$

$x+1=0$
 $-1-1$

VA: $x = -1$

domain: $x \neq 0, -1, -2$

highest: x^3

$$y = \frac{0x^3}{1x^3}$$

HA: $y = 0$