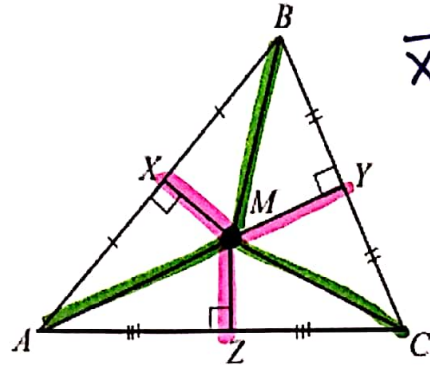# GUIDED NOTES: Centers of Triangles

Circumcenter

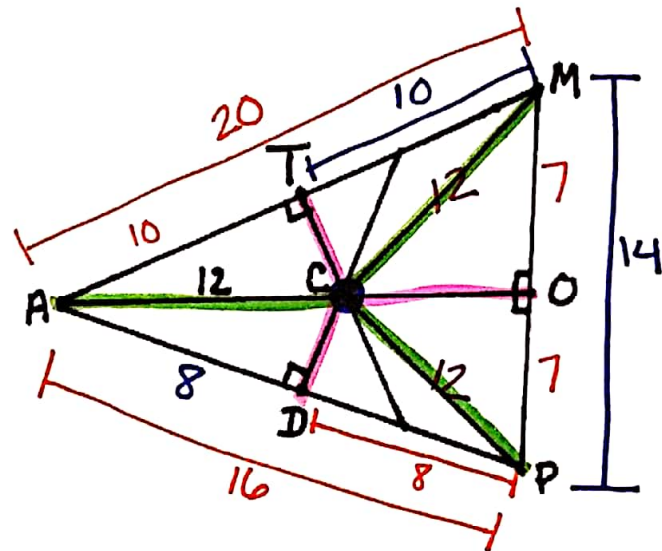$\overline{XM}, \overline{YM}, \overline{ZM}$ are perpendicular bisectors.

$\overline{MA} = \overline{MB} = \overline{MC}$

Created by: perpendicular bisectors ← make a right angle
← cut in half

Important Facts:

The _circumcenter_ is equidistant from each _vertex_ of the triangle.
↑ corner, angle

Ex1

Given	Find
C is a circumcenter.	AT = __10.__
AC = 12	CM = __12__
MP = 14	DP = __8__
TM = 10	
AD = 8	

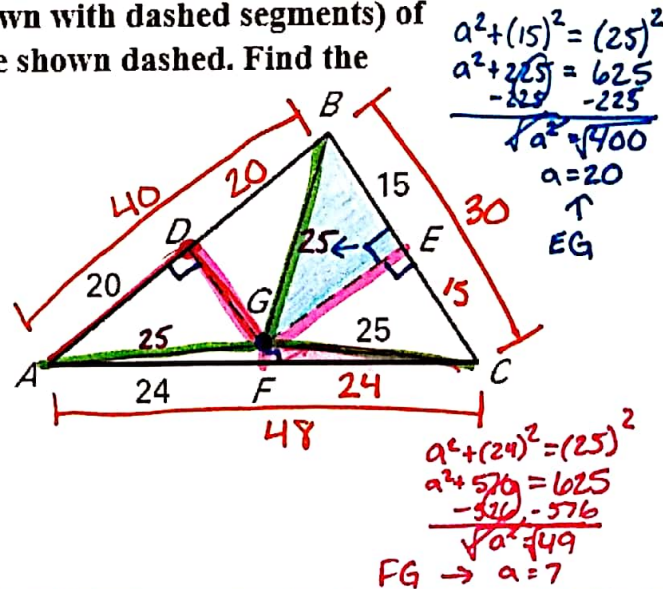Diagram labels: 20, 10, M, T, 10, 12, 7, 14, A, 12, C, O, 8, 7, D, 16, 8, P

Ex 2)

In the diagram, the perpendicular bisectors (shown with dashed segments) of △ABC meet at point G—the _circumcenter_. and are shown dashed. Find the indicated measure.

AG = __25__   FG / ~~BD~~ = __7__

CF = __24__   AB = __40__

EG / ~~CD~~ = __20__   AC = __48__
     vertex

m∠ADG = __90°__

IF BG = (2x − 15), find x.

$$2x - \cancel{15} = 25$$
$$+15 \quad +15$$
$$2x = 40$$
$$x = 20$$

Diagram labels: B, 40, 20, 15, 30, D, 25←, E, 20, G, 15, 25, 25, A, 24, F, 24, C, 48

$$a^2 + (15)^2 = (25)^2$$
$$a^2 + 225 = 625$$
$$\quad -225 \quad -225$$
$$\sqrt{a^2} = \sqrt{400}$$
$$a = 20$$
↑ EG

$$a^2 + (24)^2 = (25)^2$$
$$a^2 + 576 = 625$$
$$\quad -576 \quad -576$$
$$\sqrt{a^2} = \sqrt{49}$$
FG → $a = 7$

Incenter
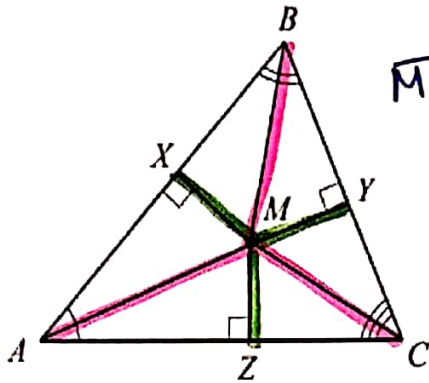
angle bisectors

$\overline{MB}, \overline{MA}, \overline{MC}$ are angle bisectors.

$\overline{MX} = \overline{MY} = \overline{MZ}$

Important Facts:

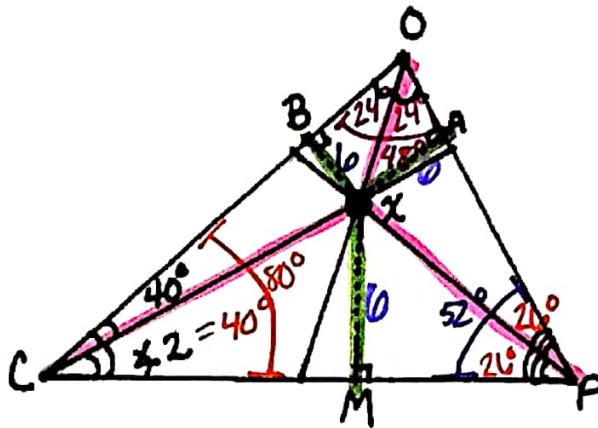The __incenter__ is equidistant from each __side__ of the triangle.

Ex 1)

X is an incenter.

BX = 6

m∠ CPO = 52°
    ↑
  vertex

$180 - 80 - 52 = 48°$

Find:

XM = __6__

m∠ 2 = __40°__

m∠ BOX = __24°__

Ex 2)

a) Find ST if S is the incenter of ΔMNP.

$a^2 + (8)^2 = (10)^2$

$a^2 + 64 = 100$

$ -64 -64$

$\sqrt{a^2} = \sqrt{36}$

$a = 6$

$ST = 6$

b) Find m∠SPU if S is the incenter of ΔMNP.

∠SPU = 31°
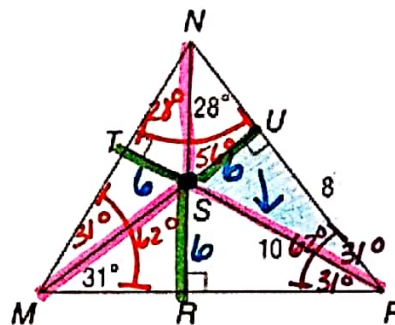
$180 - 62 - 56 = 62°$

$\overline{BZ}, \overline{AY}, \overline{CX}$
are medians.

Created by:
medians

Important Facts:
A __median__ is created
by a __vertex__ connected
to the __middle__ of
the opposite side.

$\overline{MZ} = \frac{1}{3} \cdot \overline{BZ}$
↑ short part   ↑ whole median

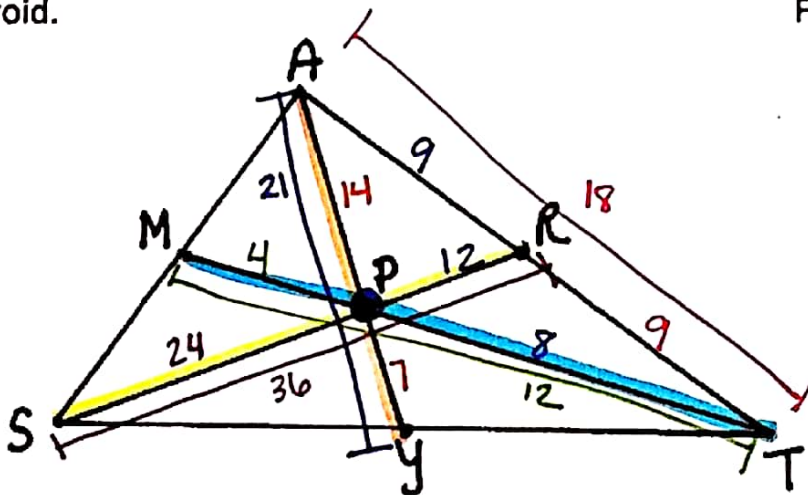$\overline{MZ} = \frac{1}{2} \cdot \overline{BM}$
↑ Short part   ↑ long part

Ex 1) *P* is a centroid.

PR = 12

PT = 8

AR = 9

AY = 21

Find:

SP= __24__

TM= __12__

AT= __18__

PY= __7__

Labels on figure: A, 21, 14, 9, 18, M, 4, P, 12, R, 24, 36, 7, 8, 9, 12, S, y, T

Ex 2) Solve for x, y, and z if N is the centroid.

$\frac{2x = 24}{2 \quad 2}$

$\boxed{x = 12}$

$\begin{array}{c} 3y + 5 = 20 \\ -5 \quad -5 \\ \hline \frac{3y = 15}{3 \quad 3} \end{array}$

$\boxed{y = 5}$

$\frac{6z = 12}{6 \quad 6}$

$\boxed{z = 2}$

Figure labels: $3y + 5 = 20$, K, O, 12, 24, $6z = 12$, M, J, 10, N, $2x = 24$, P, L