

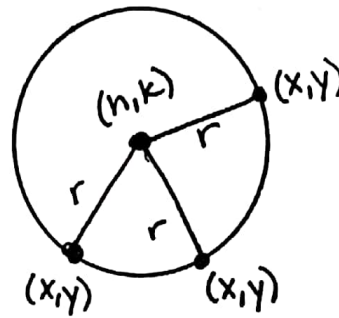
March 28

Equation of a Circle

$$(x-h)^2 + (y-k)^2 = r^2$$

center: (h, k)

radius: r



Determine center and radius:

Ex1 $(x-4)^2 + (y+2)^2 = 64$

center: $(4, -2)$

radius: $\sqrt{r^2} = \sqrt{64}$
 $r = 8$

*switch signs when identifying center!

Ex2 $(x+1)^2 + y^2 = 180$

center: $(-1, 0)$

radius: $\sqrt{r^2} = \sqrt{180}$
 $r = 6\sqrt{5}$

$$\begin{array}{c} \sqrt{180} \\ \swarrow \quad \searrow \\ 5 \quad 36 \\ \swarrow \quad \searrow \\ 6 \quad 6 \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ 2 \quad 3 \quad 2 \quad 3 \\ \hline \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5} \end{array}$$

$$\frac{3 \cdot 2 \sqrt{5}}{6\sqrt{5}}$$

Ex3 Write the equation of a circle with center $(-2, 7)$ and radius 18.

$$(x+2)^2 + (y-7)^2 = 18^2$$

$$(x+2)^2 + (y-7)^2 = 324$$

Determine center and radius:

EX4 $x^2 + y^2 + 4x - 12y + 36 = 0$

step 1: Get variables on one side of equal sign and plain numbers on the other.

$$\begin{array}{r} x^2 + y^2 + 4x - 12y + 36 = 0 \\ \underline{-36 \quad -36} \\ x^2 + y^2 + 4x - 12y = -36 \end{array}$$

step 2: Rearrange to group x's and y's together.

$$(x^2 + 4x) + (y^2 - 12y) = -36$$

step 3: Make empty shapes to complete the square in.

$$(x^2 + 4x + \boxed{4}) + (y^2 - 12y + \boxed{36}) = -36 + \boxed{4} + \boxed{36}$$

step 4: Fill in shapes. Take number in front of x and y, divide it by 2, and then square it.

$$\frac{4}{2} = (2)^2 = 4 \quad \frac{-12}{2} = (-6)^2 = 36$$

step 5: Build parenthesis. Combine plain numbers.

$$(x + 2)^2 + (y - 6)^2 = 4$$

center: $(-2, 6)$

radius: $\sqrt{r^2} = \sqrt{4}$
 $r = 2$

EX5 $x^2 + y^2 - 6x - 43 = 0$
 $\quad \quad \quad +43 \quad +43$

$$x^2 + y^2 - 6x = 43$$

$$(x^2 - 6x) + y^2 = 43$$

$$(x^2 - 6x + \boxed{9}) + y^2 = 43 + \boxed{9}$$

$$\frac{-6}{2} = (-3)^2 = 9$$

$$(x - 3)^2 + y^2 = 52$$

center: $(3, 0)$

radius: $\sqrt{r^2} = \sqrt{52}$
 $r = 2\sqrt{13}$

$$\begin{array}{c} \sqrt{52} \\ \uparrow 4 \cdot 13 \\ \begin{array}{c} 2 \cdot 2 \\ \sqrt{2 \cdot 2 \cdot 13} \\ 2\sqrt{13} \end{array} \end{array}$$