

May 13

GUIDED NOTES: Exponential Functions

An exponential function is a function in the general form $a = p(b)^t$

where $a =$ final amount

$p =$ initial amount (starting)
 Growth: $b > 1$

$b =$ growth/decay factor
 Decay: $0 < b < 1$

$t =$ number of time periods that pass

EX1. Suppose two mice live in a barn. If the number of mice quadruples every 3 months, how many mice will be in the barn after 2 years?

$$a = p(b)^t \quad 2 \text{ years} \Rightarrow 24 \text{ months}$$

- a: ?
- p: 2
- b: 4
- t: 8

$$\frac{24 \text{ months}}{3 \text{ month time periods}} = 8 \text{ time periods}$$

$$a = 2(4)^8$$

$$a = 131,072 \text{ mice}$$

EX2. The value of an iPad decreases at 35% per year. If the starting price of the iPad is \$500, how much will the iPad be worth after 5 years?

$$a = p(b)^t$$

- a: ?
- p: 500
- b: $100 - 35 = 65\% \div 100 = .65$
- t: 5

$$a = 500(.65)^5$$

$$a = \$58.01$$

When can you buy the iPad for \$5?

- $a = p(b)^t$
- a: 5
- p: 500
- b: .65
- t: ?

$$\frac{5}{500} = \frac{500(.65)^t}{500}$$

$$.01 = .65^t$$

$$\ln .01 = \ln .65^t$$

$$\frac{\ln .01}{\ln .65} = \frac{t \cdot \ln .65}{\ln .65}$$

$$10.69 \text{ years} = t$$

EX3. Suppose the acreage of forest is decreasing by 2% per year because of development. After 6 years of development, there is 4,000,000 acres of forest remaining. How many acres were originally in the forest?

- $a = p(b)^t$
- a: 4,000,000
- p: ?
- b: $100 - 2 = 98\% \div 100 = .98$
- t: 6

$$4,000,000 = p(.98)^6$$

$$\frac{4,000,000}{.89} = \frac{p \cdot .89}{.89}$$

$$4,515,475.99 \text{ acres} = p$$

EX4. Find a bank account balance to the nearest dollar, if the account starts with \$100, has an annual interest rate of 4%, and the money is left in the account for 12 years.

time period per year

$$a = p(b)^t$$

a: ?

p: 100

b: $100 + 4 = 104\% \div 100 = 1.04$

t: 12

$$a = 100(1.04)^{12}$$

$$a = \$160.10$$

If you wanted to buy a new gaming system for \$250, when will you have enough?

$$a = p(b)^t$$

a: 250

p: 100

b: 1.04

t: ?

$$\frac{250}{100} = \frac{100(1.04)^t}{100}$$

$$2.5 = 1.04^t$$

$$\ln 2.5 = \ln 1.04^t$$

$$\frac{\ln 2.5}{\ln 1.04} = \frac{t \cdot \ln 1.04}{\ln 1.04}$$

$$23.36 \text{ years} = t$$

EX5. The pesticide DDT was widely used in the United States until its ban in 1972. If the half-life of DDT is 15 years, how much DDT would be remaining after 45 years?

find b decay by 50% time period

$$a = p(b)^t$$

a: ?

p: 100

b: $100 - 50 = 50\% \div 100 = .5$

t: 3

$$\frac{45 \text{ years}}{15 \text{ year time periods}} = 3 \text{ time periods}$$

$$a = 100(.5)^3$$

$$a = 12.5 \text{ grams}$$