

May 14

GUIDED NOTES: Compound Interest

Compounded over time period:

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

Compounded continuously:

$$A = P e^{rt}$$

e is a number

A: final amount

P: initial amount (principle)

r: interest rate

n: number of times compounded in one year

t: time periods (in YEARS!)

Compounded.....	n =
yearly, annually	1
semiannually	2
quarterly	4
monthly	12

EX1. What amount will an account have after $\overset{t}{5}$ years if $\overset{P}{\$75}$ is invested at $\overset{r}{8.5\%}$ interest compounded continuously?

$$A = P e^{rt}$$

A: ?

P: 75

r: $8.5\% \div 100 = .085$

t: 5

$$A = 75 e^{.085 \cdot 5}$$

$$A = \$114.72$$

EX2. Find the amount owed at the end of $\overset{t}{9}$ years if $\overset{P}{\$5000}$ is loaned at a rate of 6% interest compounded $\overset{n=4}{\text{quarterly}}$.

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

A: ?

P: 5000

r: $6\% \div 100 = .06$

n: 4

t: 9

$$A = 5000 \left(1 + \frac{.06}{4}\right)^{4 \cdot 9}$$

$$A = \$8545.70$$

EX3. Determine the amount that must be invested at 6% interest compounded $n=12$ monthly, so that $\$200,000$ will be available for retirement in 20 years.

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$A: 200,000$$

$$P: ?$$

$$r: 6\% \div 100 = .06$$

$$n: 12$$

$$t: 20$$

$$200,000 = P \left(1 + \frac{.06}{12}\right)^{12 \cdot 20}$$

$$\frac{200,000}{3.31} = \frac{P \cdot 3.31}{3.31}$$

$$\boxed{\$60,419.23 = P}$$

EX4. What amount invested at 7% interest compounded continuously for 4 years will yield $\$700$?

$$A = Pe^{rt}$$

$$A: 700$$

$$P: ?$$

$$r: 7\% \div 100 = .07$$

$$t: 4$$

$$700 = P e^{.07 \cdot 4}$$

$$\frac{700}{1.32} = \frac{P \cdot 1.32}{1.32}$$

$$\boxed{\$529.05 = P}$$

EX5. If $\$600$ is invested at 6% interest compounded continuously, how long will it take before the amount is $\$900$?

$$A = Pe^{rt}$$

$$A: 900$$

$$P: 600$$

$$r: 6\% \div 100 = .06$$

$$t: ?$$

$$\frac{900}{600} = \frac{600 e^{.06t}}{600}$$

$$1.5 = e^{.06t}$$

$$\ln 1.5 = \ln e^{.06t}$$

$$\frac{\ln 1.5}{\ln e} = \frac{.06t \cdot \ln e}{\ln e}$$

$$\frac{.41}{.06} = \frac{.06t}{.06}$$

$$\boxed{6.76 \text{ years} = t}$$

EX6. How long does it take $\$1500$ to double if it is invested at 6% interest compounded $n=2$ semiannually?

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$A: 3000$$

$$P: 1500$$

$$r: 6\% \div 100 = .06$$

$$n: 2$$

$$t: ?$$

$$\frac{3000}{1500} = \frac{1500 \left(1 + \frac{.06}{2}\right)^{2t}}{1500}$$

$$2 = \left(1 + \frac{.06}{2}\right)^{2t}$$

$$\ln 2 = \ln \left(1 + \frac{.06}{2}\right)^{2t}$$

$$\frac{\ln 2}{\ln \left(1 + \frac{.06}{2}\right)} = \frac{2t \cdot \ln \left(1 + \frac{.06}{2}\right)}{\ln \left(1 + \frac{.06}{2}\right)}$$

$$\frac{23.45}{2} = \frac{2t}{2}$$

$$\boxed{11.72 \text{ years} = t}$$