

Unit 2 Review - Polynomials

NAME \_\_\_\_\_

**Polynomial Division**

Divide using either long division or synthetic division (when possible).

1.  $(9x^3 - 2x^2 + 5x + 4) \div (x - 3)$

$$9x^2 + 25x + 80 + \frac{244}{x-3}$$

2.  $(6x^3 + 19x^2 + 7x - 12) \div (2x + 3)$

$$3x^2 + 5x - 4$$

3.  $(12x^3 - 7x^2 - 38x + 35) \div (4x - 5)$

$$3x^2 + 2x - 7$$

4.  $(x^4 + 7x^3 - 6x + 2) \div (x + 4)$

$$x^3 + 3x^2 - 12x + 42 - \frac{166}{x+4}$$

**Remainder/Factor Theorem**

Determine which are factors of  $2x^{91} - x^{90} - 10x^{89}$ .

5.  $x - 1$

no

6.  $2x - 5$

yes

7.  $x + 2$

yes

**Polynomial Vocabulary**

Classify each polynomial by the degree and by the number of terms.

8.  $7x^3 - 2x$

cubic  
binomial

9.  $-10x^4 - 3x^3 + 2$

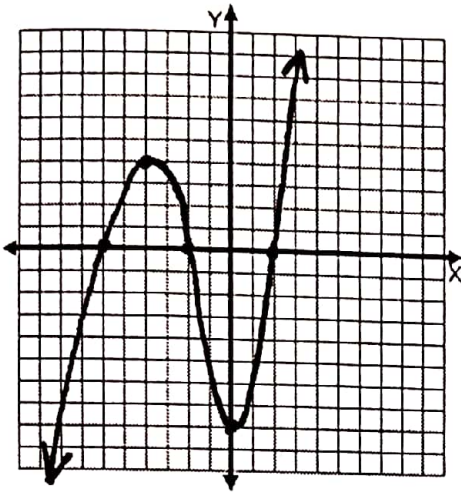
quartic  
trinomial

10. 7

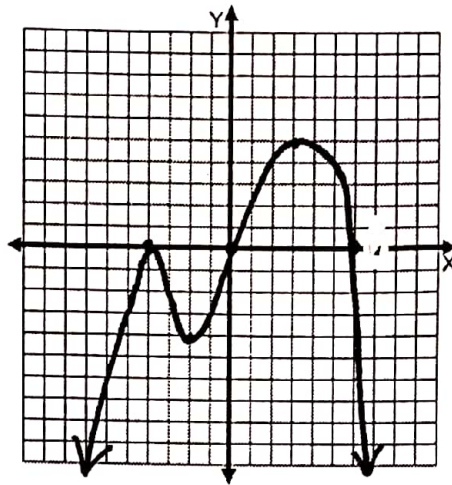
constant  
monomial

**Zeroes and Multiplicity, Extrema, Intervals for Increasing/Decreasing/Positive/Negative**  
 For each graph and equation, determine all key features.

11.



12.



Zeroes:  $x = -6 m:1, x = -2 m:1, x = 2 m:1$

Degree: 3

Extrema:  $(-4, 4)$  <sup>rela</sup>max  $(0, -8)$  <sup>rela</sup>min

Pos:  $(-6, -2)$   $(2, \infty)$

Neg:  $(-\infty, -6)$   $(-2, 2)$

Inc:  $(-\infty, -4)$   $(0, \infty)$

Dec:  $(-4, 0)$

End Behavior:  $\text{as } x \rightarrow -\infty, y \rightarrow -\infty$   
 $\text{as } x \rightarrow \infty, y \rightarrow \infty$

13.  $y = -2(x+1)^2(3x-1)$

Zeroes:  $x = -1 m:2, x = \frac{1}{3} m:1$

Degree: 3

Extrema:  $(-1, 0)$  <sup>rela</sup>min  $(-1.11, 2.11)$  <sup>rela</sup>max

Pos:  $(-\infty, -1)$   $(-1, \frac{1}{3})$

Neg:  $(\frac{1}{3}, \infty)$

Inc:  $(-1, -1.11)$

Dec:  $(-\infty, -1)$   $(-1.11, \infty)$

End Behavior:  $\text{as } x \rightarrow -\infty, y \rightarrow \infty$   
 $\text{as } x \rightarrow \infty, y \rightarrow -\infty$

Zeroes:  $x = -4 m:2, x = 0 m:1, x = 6 m:1$

Degree: 4

Extrema:  $(-4, 0)$  <sup>rela</sup>max  $(-2, -4)$  <sup>rela</sup>min  $(3, 5)$  <sup>abs</sup>max

Pos:  $(0, 6)$

Neg:  $(-\infty, -4)$   $(-4, 0)$   $(6, \infty)$

Inc:  $(-\infty, -4)$   $(-2, 3)$

Dec:  $(-4, -2)$   $(3, \infty)$

End Behavior:  $\text{as } x \rightarrow -\infty, y \rightarrow -\infty$   
 $\text{as } x \rightarrow \infty, y \rightarrow -\infty$

14.  $y = x^3(x-2)(x-3)$

Zeroes:  $x = 0 m:3, x = 2 m:1, x = 3 m:1$

Degree: 5

Extrema:  $(1.37, 2.64)$  <sup>rela</sup>max  $(2.63, -4.24)$  <sup>rela</sup>min

Pos:  $(0, 2)$   $(3, \infty)$

Neg:  $(-\infty, 0)$   $(2, 3)$

Inc:  $(-\infty, 1.37)$   $(2.63, \infty)$

Dec:  $(1.37, 2.63)$

End Behavior:  $\text{as } x \rightarrow -\infty, y \rightarrow -\infty$   
 $\text{as } x \rightarrow \infty, y \rightarrow \infty$

**Solve Polynomials**

Determine all real and complex solutions.

15.  $x^3 - 5x^2 + 3x - 15 = 0$

$$x = 5, \pm i\sqrt{3}$$

16.  $x^4 - 3x^3 - 24x^2 + 80x = 0$

$$x = -5, 0, 4, 4$$

17.  $x^3 + 64 = 0$

$$x = -4, 2 \pm 2i\sqrt{3}$$

18.  $x^3 + 5x^2 + 10x + 24 = 0$

$$x = -4, \frac{-1 \pm i\sqrt{23}}{2}$$

**Applications**

19. The weight of an ideal round-cut diamond can be modeled by  $w = 0.0074d^3 - 0.087d^2 + 0.32d$ , where  $w$  is the diamond's weight (in carats) and  $d$  is its diameter (in millimeters). According to the model, what is the weight of a diamond with a diameter of 12 millimeters?

4.0992 carats

20. The profit  $P$  (in millions of dollars) for a t-shirt manufacturer can be modeled by  $P = -x^3 + 5x^2 + 9x$ , where  $x$  is the number of t-shirts produced (in millions). Currently, the company produces 5 million t-shirts and makes a profit of \$45,000,000. What lesser number of t-shirts could the company produce and still make the same profit?

3 million t-shirts

21. A box has a height of  $x - 4$  inches and a length of  $x + 3$  inches. If the volume of the box is  $2x^3 - 3x^2 - 23x + 12$  cubic inches, determine the width of the box.

$$2x - 1$$

22. When fighter pilots train for dog-fighting, a "hard-deck" is usually established below which no competitive activity can take place. The polynomial graph given shows Maverick's altitude ( $y$  in 100s of feet) above and below this hard-deck during a 5 second ( $x$ ) interval.

a. What is the lowest possible degree of this polynomial?

4

b. How many total seconds was Maverick above the hard-deck during the first 5 seconds?

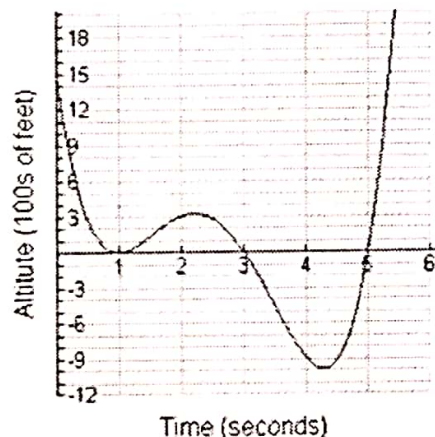
3 seconds

c. After how many seconds is Maverick 300 feet above the hard-deck?

3.5 seconds

d. Determine the equation of the function in factored form.

$$(x - 1)^2(x - 3)(x - 5)$$



### Rates of Change

23. Find the average rate of change from  $x = -1$  to  $x = 3$  for each of the functions below.

a.  $a(x) = 2x + 3$

2

b.  $b(x) = x^2 - 2$

2

c.  $c(x) = 2^x - 1$

1.875

d. Which function has the greatest average rate of change over the interval  $[-1, 3]$ ?

$a(x)$  and  $b(x)$

24. In general as  $x \rightarrow \infty$ , which function eventually grows at the fastest rate?

a.  $a(x) = 3x$

b.  $b(x) = x^3$

c.  $c(x) = 3^x$